## Fractallike structures and the strongly kicked H-atom

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We investigate the classical 1-dim. model of H-atoms (1) prepared in extremal Stark-Rydberg states which are exposed to strong periodic electromagnetic kicks (cf.2). An ensemble of initial phase-space points with defined energy decays in time under the effect of the kicks according to a power law. The underlying mechanism also allows an understanding of fractallike patterns made up by the regions of equal stability under the kick-pulses.

Using convenient units the model-Hamiltonian is written:

$$H = p^2/2 - 1/q - \beta \cdot q \sum_k \delta\,(t - k \cdot \tau)$$

The (p,q)-coordinates immediately give the kickmap: $p' = p + \beta$ , $q' = q$ , whereas the unperturbed Kepler-motion between two kicks, in the following called 'twist' (cf.3), is described canonically in action-angle variables $(n, \theta)$.

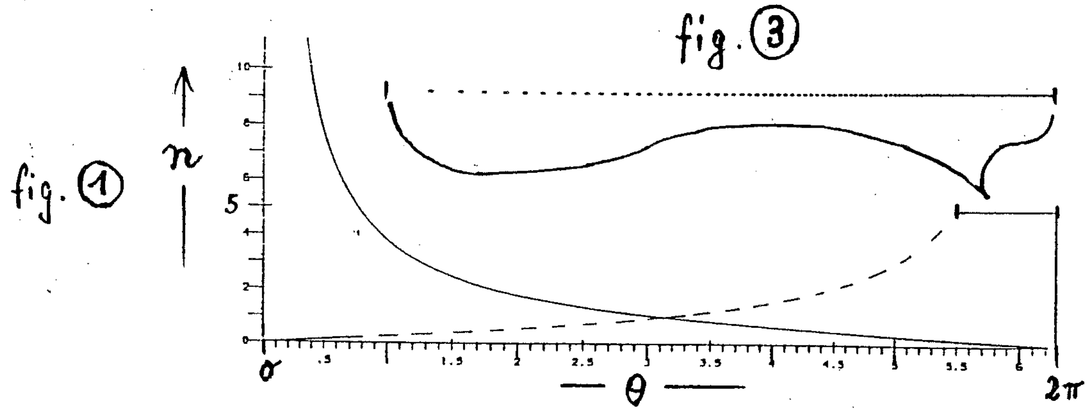
For every starting point in phase-space the system remains bound under a number of kick-pulses characteristic of that point. We have assigned the same colour to all points which ionize after an equal time, thus making visible the 'equiionizing sets'. The speed of ionization turns out to fluctuate strongly. The pattern of the 'equiionizing sets' shows structure on all scales. One might be led to the conclusion that the ionization mechanism is analogous to the 'recursive removal of the middle-third' which brings about the famous Cantor set. But our complex structure of phase space arises already after very few kicks - which act as removing steps -, whereas the middle-third process only generates structure down to scales that are limited by the number of recurrences already carried out. The mentioned power-law decay of a phase-space ensemble is also in contrast with the behaviour of Cantor-processes: Removing a constant fraction of the surviving set at every stage results in an exponential decay.

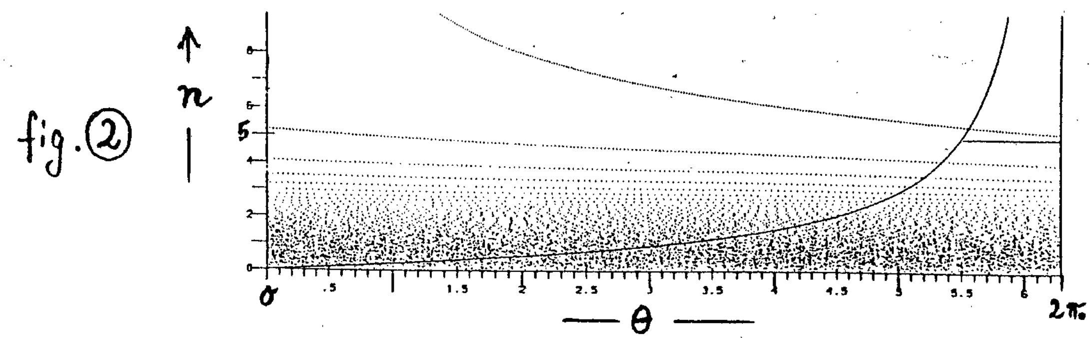
To understand these peculiar features we have to look closer at the kick- and twist-maps:

Fig.1:*Result of one kick exerted on the line at* $n = 5$.

Fig.2:*Result of one twist after the kick*

Fig.3:*Ionization pattern, impressed on the original interval at* $n = 5$ *, by the sequence kick – twist – kick.*




Figures 1 and 2 contain in addition a $tan\,(\theta/4)$- curve. Points to its left are ionized by the next kick.

The singular behaviour of the Kepler-twist at $n = 0$ accounts for the immediate arising of structure at all scales (see figure 3). To give a decay as $1/t^{\gamma}$ the fraction of ionizing points has to decrease as $\gamma/t$ , t denoting discrete time steps. This is accomplished by a continuous wandering of the (initially $\delta$ - shaped ) n-distribution towards lower values, where the effect of the twist can be modelled by homogenizing the $\theta$ - distribution of phase-space points. Thus a simple comparison with the lowest non-ionizing $\theta$-angle for given n, $\theta = 4 \cdot arctan\,(n)$, gives the right behaviour. The stochastic effect of the twist also explains the numerically observed insensitivity of the decay-power against variations of the kick-frequency.

Summarising, we have found that a recursive mechanism which is only structurally, but not quantitatively constant from step to step is responsible for a power-law-decay.

1) R. V. Jensen, Phys. Rev. Lett. 49, 1365 (1982)
2) R. Blümel, U. Smilansky, Phys. Rev. A 30, 1040 (1984)
3) G. Troll, U. Smilansky, Physica D, in press