Nonuniversality of the localization length in the symplectic kicked rotor model

M. Thaha and R. Blümel
Department of Physics
University of Delaware, Newark, Delaware 19716
(October 13, 1993)

In 1989 R. Scharf introduced the symplectic kicked rotor [1]. We specialized his general Hamiltonian to a Hamiltonian containing 5 control parameters

$$\begin{aligned} H = \frac{1}{2}\tau\hat{l}^2 + k[\cos(\pi p/2)\cos(\pi q/2)\cos(\theta + r) \\ + \frac{1}{2}\cos(\pi p/2)\sin(\pi q/2)\sin(2\theta)\sigma_x \\ + \sin(\pi p/2)\sin(\theta)\sigma_z]\delta_p(t) \end{aligned} \quad (1)$$

and studied its localization length. The control parameters τ and k are familiar from the standard kicked rotor [2], θ is a rotation angle, $\hat{l} = -i\partial/\partial\theta$ is the angular momentum operator, $\sigma_j, j = x, y, z$ are the Pauli matrices and $\delta_p(t)$ is the 1-periodic δ function. The parameters p, q and r allow us to switch between the three universal symmetry classes, orthogonal, unitary and symplectic, respectively. For $r = 0$, $K = 10$, $\tau = 0.05 \times 2\pi/g$, $g = (\sqrt{5}-1)/2$, the location of the three symmetry classes are shown as the plateau regions in Fig.1 as a function of $x = \ln(p/(1-p))$ and $y = \ln(q/(1-q))$.

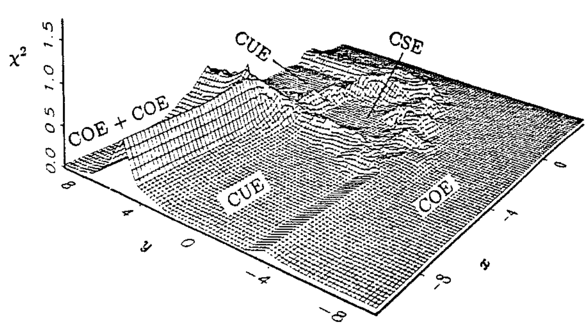

FIG. 1. Symmetry plateaus of the symplectic kicked rotor defined in (1).

Fig.1 was obtained in the folowing way. Defining $P_{x,y}(s)$ as the nearest neighbor spacing distribution of the quasi energies of the one-cycle propagator U of (1), the vertical axis in Fig. 1 represents the minimum of the four χ^2 values obtained when $P_{x,y}(s)$ is compared to the known probability distributions of the three circular ensembles (COE, CUE, CSE) and the nearest neighbor spacing distribution for a 50%-50% mixture of two statistically independent COE ensembles.

The line $y = 1$ in Fig.1 crosses three symmetry plateaus in the following order: CUE $\rightarrow$ CSE $\rightarrow$ COE. Denoting by $|\alpha>$ the quasi energy states of U, we define the localisation length of an l state in the quasi energy basis as the participation ratio $\xi(l) = 1/\sum_\alpha |<l|\alpha>|^4$. On the line $y = 1$ three different universality classes are encountered. In order to be able to compare meaningfully the localization lengths of l states corresponding to different universality classes, the purely dynamical effects of the nonlinear system (1) (arising from a change of the control parameters p, q, and r, respectively) have to be taken into account. Since in strongly localizing quantum chaotic systems the localization length is proportional to the classical diffusion constant [2,3], effects exclusively due to symmetry are contained in the quantity $c = \xi/D$ where D is the classical diffusion constant. Methods for calculating D for the spin-1/2 system (1) were recently developed [4]. Fig.2 shows the normalized proportionality constant $\hat{c} = c(x, y = 1)/c(x = 5, y = 1)$ as a function of x on the line $y = 1$.

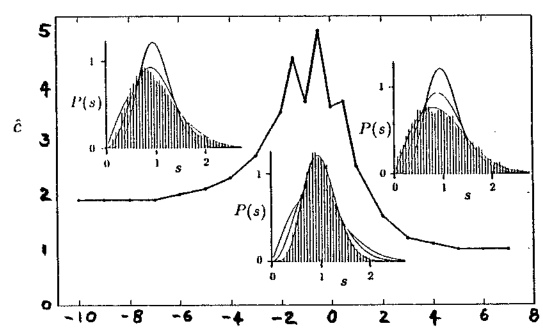

FIG. 2. Normalized localization length $\hat{c}$ as a function of x on the path $(x, y = 1)$. The three different symmetry classes encountered on the path are indicated by the histogram insets which correspond to $x = -8$, $x = -1$ and $x = 5$, respectively.

Fig. 2 shows that the localization length is roughly constant within a well developed symmetry region. It grows by a factor 2 in the transition region between the CUE and CSE regions and reduces by a factor 4 in the transition region between the CSE and COE regions, respectively. This result is consistent with the prediction by Pichard et al. [5].

Switching on the r parameter, the Kramers degeneracy of (1) is broken. We chose the path $\Gamma = (x = -1, y = 1, r)$ to investigate the effects of Kramers degeneracy breaking on the localization length. A CSE $\rightarrow$ CUE $\rightarrow$ COE transition occurs on Γ as a function of increasing r. In analogy to Fig. 2 we define the normalized localization length $\gamma(z) = c(x = -1, y = 1, z)/c(x = -1, y = 1, z = 0)$. The result is displayed in Fig. 3.

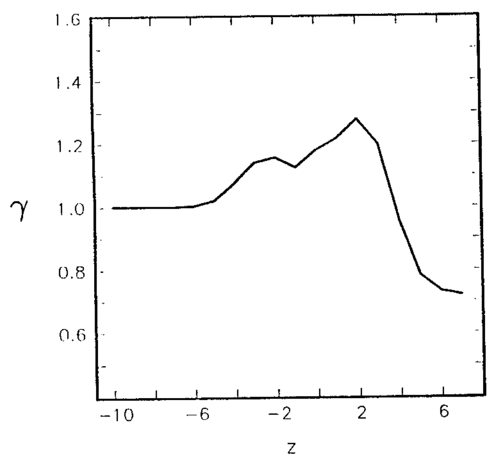

FIG. 3. Normalized localization length γ as a function of the symmetry breaking parameter z.

If the result displayed in Fig. 2 were universal, we would have expected to see a factor 2 *decrease* of the localisation length when entering the CUE region on the path Γ. According to the results displayed in Fig. 3 this is not the case indicating that the localization length may not be a universal function of the symmetry class. Kramers degeneracy plays an independent role in determining the localisation length. In order to strengthen our results we have to (i) calculate and evaluate the localization length of the quasi energy states in the l basis and (ii) confirm our results on additional paths in the (p, q, r) parameter space.

We are currently investigating the question whether the universal factors 2 and 4 (universal in Kramers degeneracy conserving systems) can be seen in the localization lengths of other quantum chaotic systems such as Rydberg atoms in strong radiation fields.

This research was conducted using the Cornell National Supercomputer Facility, a resource of the Center for Theory and Simulations in Science and Engineering (Cornell Theory Center), which receives major funding from the National Science Foundation and IBM Corporation, with additional support from New York State and members of the Corporate Research Institute. R. B. is grateful for financial support from the University of Delaware Research Foundation and the Deutsche Forschungsgemeinschaft.

[1] R. Scharf, J. Phys. **A** 22, 4223 (1989).
[2] F. M. Izraelev, Phys. Rep. **196**, 299 (1990).
[3] G. Casati, B. V. Chirikov, I. Guarneri, and D. L. Shepelyansky, Phys. Rep: **154**, 77 (1987).
[4] M. Thaha, R. Blümel, and U. Smilansky, Phys. Rev. **E** 48, 1764 (1993).
[5] J.-L. Pichard, M. Sanquer, K. Slevin, and P. Debray, Phys. Rev. Lett. **65**, 1812 (1990).